

Filtering of the absolute value of photon-number difference for two-mode macroscopic quantum states

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We propose a filter that selects two-mode high number Fock states whose photon-number difference exceeds a certain value. Such a filter is important for the engineering of macroscopic quantum states of light and for the control of bright light beams. It improves distinguishability of some states but preserves macroscopic superpositions. We suggest an operational implementation of such a filter. To this end we employ an interference effect in which two input ports of a beamsplitter are entered by highly populated multi-photon states. If the photon numbers at the two input ports are almost equal, the photons are highly asymmetrically distributed between the two outputs. If photon numbers differ a lot, the exit ports get nearly equal photon occupation numbers.

The set of quantum states that are efficiently produced is limited. Nevertheless, with quantum state engineering one can modify or enhance certain properties of experimentally accessible states. In particular, measurement induced state operations allow one to filter out states of required properties. In the case of intensity measurement, of considerable help are threshold detectors, selecting Fock states or superpositions with sufficiently high population numbers. Examples of low-threshold detectors are known: single photon on-off detectors and human eyes [1, 2]. Such detectors can be utilized in feed-forward techniques based on tap measurements, in which effectively POVM measurements [3] are performed leading to quantum operations. As a result, it is possible to block light of unwanted properties (too low or too high intensity). More complicated filters for Fock states are based on the interference effects [4, 5].

A more challenging task is to construct a filter that would select states with certain properties without destroying quantum superpositions. This is especially important for macroscopic superpositions of Schrödinger-cat type, which suffer from low distinguishability and are easily destroyed by inefficient detection [6–8]. Such macroscopic superpositions combine quantum properties with macroscopic population and could enable efficient light-matter coupling, and therefore are of much interest for various protocols in quantum information technology, such as quantum memory [9, 10], quantum key distribution [11], macroscopic Bell inequality tests [12, 13], and quantum metrology [14].

Here, we propose a blueprint of such a device which is further called a modulus of intensity difference fil-

ter (MDF). It filters out two-mode states of light whose mode populations differ by more than a certain threshold. Its optical implementation will be based on the interference of multi-photon states entering a beamsplitter, each mode via a different input port. Theoretically, the aimed filter should perform the projection operation

$$\mathcal{P}_{\delta_{th}} = \sum_{k,l=0; |k-l| \geq \delta_{th}}^{\infty} |k, l\rangle \langle k, l|, \quad (1)$$

where $|k, l\rangle$ is a two-mode Fock state. Further on, without the loss of generality, we will assume that the two modes of the initial state are distinguished by orthogonal polarizations, while all other characteristics are identical. For $\delta_{th} > 0$ the filter acts as “quantum scissors” [15]. It cuts out those terms of an expression of the state in the Fock basis for which the occupation difference is below the threshold and preserves those where the difference is above it.

We would like to comment on the two key features of such a filter. It estimates the absolute value of the difference instead of the difference itself. This has an advantage. Since the spectrum of the operator $\mathcal{P}_{\delta_{th}}$ is fully degenerate, the filter does not provide any information on which mode was more populated. Further on we discuss the action of a MDF on a macroscopic qubit encoded in polarization (see formula (3)). We show that it is not able to distinguish the states and filters them fairly. Secondly, the estimation is performed in a “yes”-“no” manner: the exact value of the modulus is *never* measured and thus, this information *does not exist*. This property is of great importance for all quantum protocols that require state engineering preserving the superposition. This is why we

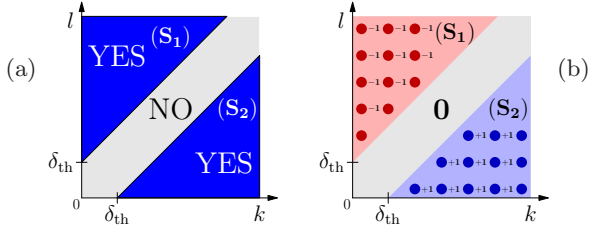


FIG. 1: (Color online) Comparison of two filtering techniques: absolute difference (MDF) (a) and orthogonality filter (b).

call this device a filter.

These two features are the main difference between the MDF and the orthogonality filter (OF) based on intensity difference measurements [16]. OF is the basic element in the setups performing measurement induced operations on the macroscopic polarization states [12]. It splits the state in two orthogonal polarizations on a PBS and measures their intensities I and I_\perp by photon multipliers (PM) separately. This is a von Neumann measurement $\pi_{kl} = |k, l\rangle\langle k, l|$ and it projects the state on a single Fock state. Next, the electronic signals from PMs are passed to the OF and electronic “dichotomization” is applied. The intensity difference is calculated by taking the signals difference and it is compared with a threshold value. The $+1$ (-1) value is signaled if $I - I_\perp > \delta_{th}$ ($I_\perp - I > \delta_{th}$), and 0 otherwise. Thus, this is a comparative threshold measurement. Contrary to MDF, such a scheme destroys superpositions. Measurement induced operations on the multiphoton states based on the OF device allow only for efficient state discrimination, not filtering. Thus, OF is not suitable for preselection strategies in Bell tests, of the kind suggested in [12]. There, polarization analysis on the reflected beam leads to state identification in the transmitted beam and breaks the entanglement. The action of MDF and OF is compared in Fig. 1. The former projects onto S_1 and S_2 areas simultaneously and all superposition terms belonging to them are preserved. The latter combined with photon multipliers projects the state on a Fock state either in S_1 or S_2 (red or blue dot in the figure).

Let us analyze the action of the MDF on specific “macroscopic qubits,” produced by the optimal phase covariant quantum cloners in the process of phase sensitive coherent parametric amplification [2, 16, 17],

$$\begin{aligned}
 |\Phi\rangle &= \sum_{i,j=0}^{\infty} \gamma_{ij} |(2i+1)_\varphi, (2j)_\varphi^\perp\rangle, \\
 |\Phi_\perp\rangle &= \sum_{i,j=0}^{\infty} \gamma_{ij} |(2j)_\varphi, (2i+1)_\varphi^\perp\rangle,
 \end{aligned} \quad (2)$$

where φ and φ^\perp are the two orthogonal polarizations from the equatorial plane of the Poincaré sphere, the real-valued probability amplitude $\gamma_{ij} =$

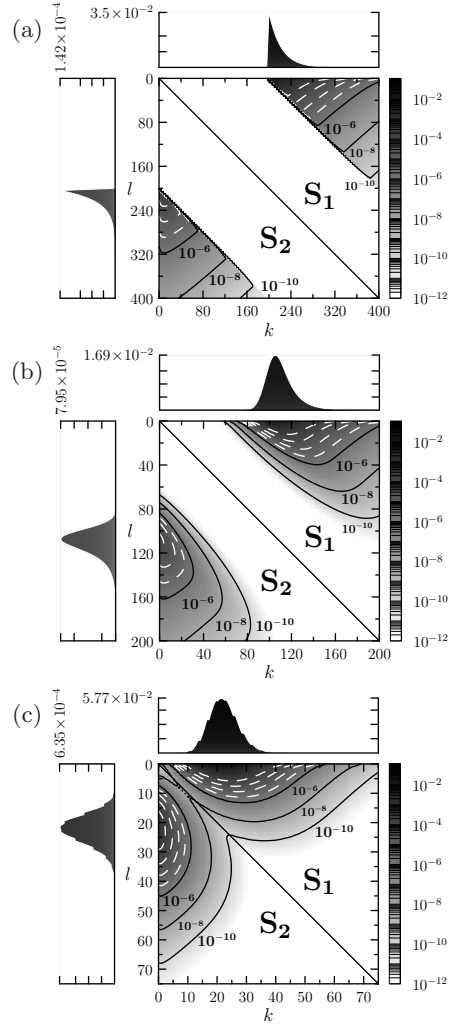


FIG. 2: Distribution p_Φ computed for $g = 1.87$ and filtering threshold $\delta_{th} = 200$ with 0% (a), 50% (b), 90% (c) losses.

$\cosh g^{-2} ((\tanh g)/2)^{i+j} \sqrt{(1+2i)!(2j)!}/i!/j!$, g is the parametric gain. Please also note that, due to the different parity of occupation in the Fock basis, $|\Phi\rangle$ and $|\Phi_\perp\rangle$ are orthogonal and thus form a basis for a macroscopic qubit. In a recent experiment [16], each pulse contained up to $4 \sinh(g) \simeq 10^4$ photons on the average. However, in such a photon number regime, detectors are not single photon resolving, but distinguish counts varying by ± 150 photons in the best case [17]. This makes the macroqubits hardly distinguishable through direct detection [16]. Distinguishability may be quantified in terms of the photon distribution $p_\Phi(k, l) = |\langle k, l | \Phi \rangle|^2$ and $p_{\Phi_\perp}(k, l)$ giving the probabilities of finding simultaneously k photons in φ and l in φ^\perp

$$p_\Phi(k, l) = \sum_{i,j=0}^{\infty} \gamma_{ij}^2 \delta_{k,2i+1} \delta_{l,2j}, \quad p_{\Phi_\perp}(k, l) = p_\Phi(l, k), \quad (3)$$

where the Kronecker delta $\delta_{a,b}$ equals 1 if $a = b$ or 0

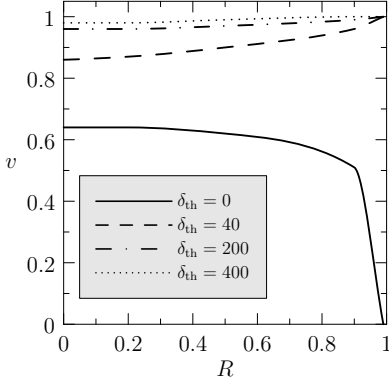


FIG. 3: Distinguishability v evaluated for the gain $g = 1.87$ and different threshold values δ_{th} as a function of the losses R .

otherwise. Distinguishability is given by

$$v = P_{\Phi}^{(S_1)} - P_{\Phi_{\perp}}^{(S_1)} = P_{\Phi}^{(S_1)} - P_{\Phi}^{(S_2)}, \quad (4)$$

where $P_{\Phi}^{(S_i)} = \sum_{k,l \in S_i} p_{\Phi}(k, l)$ is the probability of finding $|\Phi\rangle$ in S_i of Fig. 1 and $P_{\Phi}^{(S_1)} + P_{\Phi}^{(S_2)} = 1$. It increases if $|\Phi\rangle$ ($|\Phi_{\perp}\rangle$) starts to occupy mostly one of S_i regions, e.g. S_1 (S_2), with increasing δ_{th} . Fully distinguishable (indistinguishable) states have $v = 1$ ($v = 0$).

Our analysis is performed taking into account possible losses, which are modeled by a BS with reflectivity R giving the loss. The p_{Φ} distributions evaluated for $g = 1.87$ and $\delta_{th} = 200$ in the presence of 0%, 50% and 90% of losses are depicted in Fig. 2. The filtering results in cutting out a stripe $\sqrt{2}\delta_{th}$ wide located symmetrically along the $k = l$ line. The state $|\Phi\rangle$ occupies two disjoint regions of space: the top (S_1) and bottom (S_2) triangles, but increasing the threshold reduces the contribution of p_{Φ} in S_2 : the peak value goes down originally from 10^{-2} to $2 \cdot 10^{-4}$ for $\delta_{th} = 200$ and $R = 0$. At the same time the distribution peak in S_1 increases from 10^{-2} to $4 \cdot 10^{-2}$. Similar behavior is observed for higher gains. Behavior of $p_{\Phi_{\perp}}$ is identical but mirror reflected. Thus, distinguishability increases. The particle loss results in shifting the distribution towards the origin of the coordinates, i.e. vacuum state. The distribution peaks become smooth and symmetric. The edges along the threshold lines are blurred and the bigger losses are, the smaller width of the gap is. It disappears completely for 90% of losses. With increasing losses the height of the upper and left peak first, drops due to the fact that the distribution “melts” and next, increases because the total probability over the shrinking area has to be 1.

For states (2) we computed their distinguishability v for several filtering thresholds δ_{th} as a function of losses, see Fig. 3. If no filtering is applied, the distinguishability $v = 0.64$ is independent of the gain value and losses, but drops quickly to 0 if $R > 0.9$. If δ_{th} increases, v increases

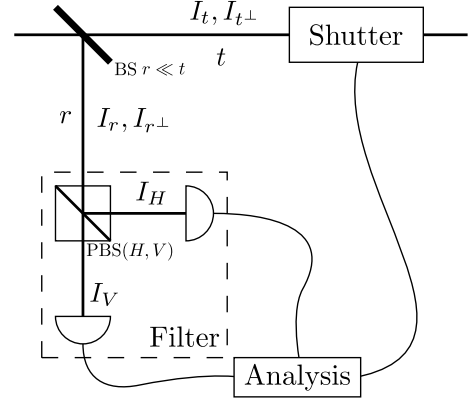


FIG. 4: An implementation of modulus of intensity difference filter. The description is in the main text.

as well and approaches unity with reasonable probability, e.g. $v = 0.96$ with the probability of success $p_s = 10^{-4}$. Obviously, for $R = 1$ we get $v = 0$.

A physical implementation of an MDF can be realized by a tap measurement, interference of Fock states on a beamsplitter and the feed forward technique, see Fig. 4. Let us consider a superposition of two mode polarization Fock states. The tap measurement is realized by a highly biased BS with low reflectivity r , e.g. $r = 10\%$, which reflects only a small fraction of the state leaving the transmitted part almost unchanged. The analysis of probability distribution for a BS shows that, for highly populated input with super-Poissonian statistics, such BS should not reflect more than 20% of the input intensity to preserve most of the character of the initial state in the transmitted beam. Under such conditions, an estimation of the population difference for the reflected beam will give us an estimation for the transmitted beam. One should employ here the “worst” case assumption. The reflected part will be filtered via an interference and measurement process, and the measurement result will determine a feed forward action on the transmitted beam, which will be opening or closing of a shutter.

Let us describe in detail the filtering setup. The annihilation operators of the two polarization modes of the reflected beam will be denoted as $a_r, a_{r\perp}$. The corresponding intensities are given by $\mathcal{I}_r = a_r^\dagger a_r$, $\mathcal{I}_{r\perp} = a_{r\perp}^\dagger a_{r\perp}$. They enter the input ports of the polarizing beamsplitter (PBS). We shall assume that the PBS is oriented in linear polarization HV basis with the output operators being $a_H = 1/\sqrt{2}(a_r + a_{r\perp})$, $a_V = 1/\sqrt{2}(a_{r\perp} - a_r)$. The two exit beams, polarized H and V, propagate to a pair of detectors, which measure the intensities. If at a given moment the respective detectors’ readings give photon occupation numbers K and L , this is equivalent to a collapse of the state into $|\Psi\rangle_{out} = |K, L\rangle = 1/\sqrt{K!L!} a_H^\dagger^K a_V^\dagger^L |0\rangle$. This state, according to the aforementioned mode trans-

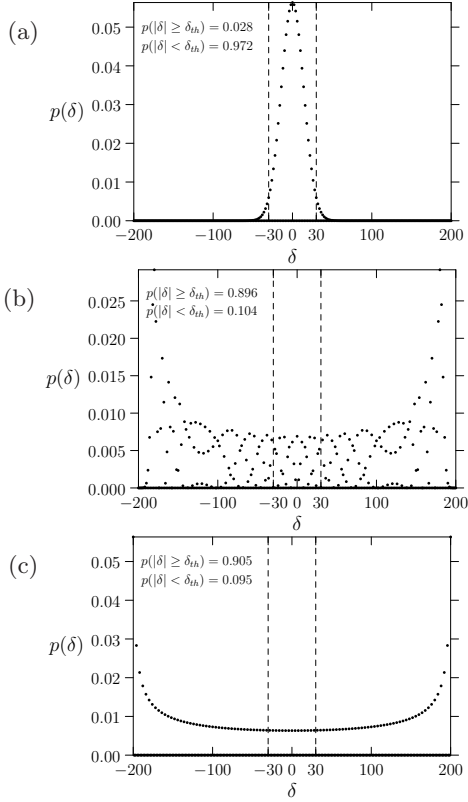


FIG. 5: Distribution of the population difference $p(\delta)$ in the input state with $s = 200$ photons conditioned on the measurement of $K = 0$, $L = 200$ (a), $K = 60$, $L = 140$ (b), $K = L = 100$ (c) at the PBS output.

formation at our PBS can be rewritten in the terms of the input modes as the following superposition [18]

$$|\Psi\rangle_{in} = \frac{1}{\sqrt{2^{K+L}}\sqrt{K!L!}} \sum_{p_1=0}^K \sum_{p_2=0}^L \binom{K}{p_1} \binom{L}{p_2} (-1)^{p_2} \frac{\sqrt{(p_1+p_2)!} \sqrt{(K+L-p_1-p_2)!}}{|p_1+p_2, K+L-p_1-p_2\rangle_{r,r^\perp}}. \quad (5)$$

The analysis of the superposition shows that if the output photon numbers K , L are measured and the cases where they differ little are selected, then, with a high probability, the input photon numbers would differ by more than a certain threshold. *Vice versa*, if K and L differ a lot this implies small differences in the occupation numbers of the input modes, see Fig. 5, where characteristic examples are given. In this way we can filter out the states for which the photon-number difference exceeds a certain threshold value. Note that the states in terms of input modes occupation numbers are not Fock ones, but superpositions of them.

Fig. 5 shows the probability distribution $p(\delta)$ of the population difference in the input state $\delta = K + L - 2p_1 - 2p_2$ for given K and L and the total photon num-

ber $s = 200$. We also calculated the probabilities that $|\delta|$ was below or above a given δ_{th} (vertical dashed lines $\delta_{th} = 30$). The filter works probabilistically. For any outcome K and L all values of δ are possible, but not equally probable. For the majority of the cases, for a reasonable threshold the probability of high population difference is higher than the probability of the low one. If the probability is satisfactory e.g. above 90%, as is for $K = L = 100$, the feed forward signal opens a shutter in the transmitted beam of the initial BS, Fig. 4. Otherwise, we reject it and the shutter remains closed.

The proposed scheme of modulus of intensity difference filter is feasible. This is a threshold measurement which projects the input state on a certain multi-dimensional subspace of the Hilbert space, thus enabling quantum superpositions and further state processing. The filter employs the interference of multi-photon states on a beam-splitter and it works for any two-mode polarization state with super-Poissonian statistics. This property is important due to the properties of the tapping BS. Realization of such measurements with commonly used photon detectors is quite challenging, however properties of the MDF are worth the possible effort. The filter could be applied in the engineering of macroscopic states of light. In the case of macro-qubits it circumvents the problem of inefficient detection, and improves their distinguishability, thus making them good quantum information carriers for quantum information protocols and quantum metrology.

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